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# Shock waves

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The first volume of the *Journal of Fluid Mechanics* contained nine articles (of 39) on shock waves. Some of these pioneered new branches of fluid mechanics. Others dealt with older problem areas. Surprising is one's realization that important elements of all topics are still of current interest. The subjects treated were shock structure, diffraction, refraction, waves in supersonic and hypersonic flows, large-amplitude acoustic and blast waves, and astrophysical processes. The subsequent addition of work on chemically reactive flows, radiating and laser-induced shocks, the effects of electric and magnetic fields on shock propagation in ionized media and the development of computer-based methods of analysis have greatly broadened the scope of shock wave investigations during the ensuing twenty-five years.

The paper traces some of the principal lines of investigation from early motivations to the present state of understanding and application. Motivation is not often consciously expressed in the scientific literature. Usually an external motivation in terms of identifiable needs for better understanding for the solution of practical problems can be identified; though much excellent work must be ascribed to that ubiquitous trait curiosity.

The topics covered in this article were chosen as representative of the basic elements of shock wave interactions and effects. They are: shock structure, refraction, diffraction, shocks in liquid helium, and condensation and liquefaction shocks. The paper closes with an assessment of how approximate and computational methods developed for handling complex flow problems fare when applied to some of the basic shock interactions considered here. Most of the emphasis will be on shock waves in gases, for which knowledge of an equation of state has been key to the significant advances made during the last twenty-five years. For liquids and solids, shock waves have been used the other way around; to study state properties.

## 1. Introduction

The occurrence of shock waves is commonly associated with supersonic flight, explosions and electric discharges. Other processes have also been found to generate abrupt compression fronts having similar features. The importance of shock waves in practical applications of fluid mechanics is due to the nearly instantaneous changes in fluid velocity and pressure they produce, while the inherent nonlinearity of the governing equations provides a challenging field for analysis. Thus the geometrical design of any structure associated with supersonic flows is directly influenced by the fact that compression may occur over an infinitesimal interval but expansion must occupy a substantial space. The search for adequate methods to treat the nonlinear terms in the fluid flow equations has unfolded a rich array of analytical solutions, series approximations, special methods such as asymptotic matching and, most recently, numerical techniques. We find examples where the accuracy of an approximation is far better than the underlying assumptions seem to warrant and, by contrast, instances where an apparently clear and appropriate set of assumptions yield highly misleading results. Such anomalies have helped to keep the study of shock waves alive and well.

Shock waves appear when elements in a fluid approach one another with a velocity larger than the local sound speed. The thickness of a shock front in air is typically $10^{-7}$ m, very small compared to other lengths characteristic of most fluid flows. Only compression shocks are observed; rarefaction shocks imply a net entropy decrease for any medium having credible thermodynamic properties. In the weak limit a shock becomes an infinitesimal sound signal with pressure–temperature–density relations given by acoustic theory. Short of relativistic velocities, no upper limit exists on the possible speed, pressure rise, or temperature rise of a shock wave although the density ratio across a shock front has an asymptotic value dictated by the molecular properties of the fluid.

This article traces the development of interest and understanding about a number of important shock wave processes. A surprising number of the problems of current fundamental and practical concern were the subject of papers in the first volume of this journal. Nine of the 39 articles appearing there dealt with shock structure, diffraction, refraction, waves in supersonic and hypersonic flow, large amplitude acoustic and blast waves and astrophysical processes. Subsequent events revealed the role of radiation in shock formation and propagation and the rich field of electro- and magnetogasdynamic waves.

In preparing the article it became apparent that much of the highly original and fundamental work on shock waves has been stimulated by practical needs. Therefore, rather than attempting either a definitive historical survey or comprehensive coverage of the entire field, both of which would be beyond my competence anyway, I have chosen to trace a modest number of aspects of the subject from their origins to their current status. The references given are intended to highlight key features of development. Readers with special interests may use the citations in these papers to recover more complete details. Much of the following account treats shock waves in gases where the knowledge of an equation of state makes possible something of a parity between theoretical and experimental understanding.

The following sections deal in turn with: shock structure, regular and Mach reflection, refraction, diffraction, shock waves in helium I and II, condensation and liquefaction shocks, and numerical methods.

## 2. Structure of shock waves

Riemann's well-known theory for the steepening of compression waves and Schardin's early work with spark photography indicated that shock waves must be very thin compared with ordinary laboratory dimensions. A simplified model in which viscous dissipation of kinetic energy is balanced by convective steepening yielded a value for shock thickness of the order of a few molecular mean free paths, or $\sim 10^{-7}$ m in air. Gilbarg & Paolucci (1953) were the first to solve the complete Navier–Stokes equations applicable to this problem. They found good agreement between the continuum theory for an ideal gas and the available experimental profiles for weak shocks, $M < 1{\cdot}5$, but the thickness predicted for stronger shocks was less than one mean free path in the gas ahead of the shock front. For such large gradients the applicability of continuum theory should be suspect despite the observed agreement.

Since the equations of fluid mechanics correspond to the zeroth- and first-order terms in the kinetic-theory expansion of the Boltzmann velocity distribution function, more accurate results for strong shocks were expected when higher-order terms were included. Early results were disappointing. In general the Navier–Stokes solutions persisted in fitting the data best and difficulties were encountered in getting the kinetic-theory solutions to converge even for shock Mach numbers below 2. Grad (1952), for instance, was able to obtain solutions only up to $M_s = 1{\cdot}65$ using a method based on thirteen integrals (moments) of the velocity distribution function that accounted for density, temperature, velocity, stress and heat flow. Except for the important conclusion that shock fronts are indeed just one or a few mean free paths thick, and therefore could be treated as discontinuities in most situations, theoretical and experimental understanding remained in limbo for a long time. We shall see presently how the matter was finally resolved. Meanwhile another aspect of shock structure attracted major attention.

Blackman's (1956) paper in volume 1 of this Journal described the role of molecular vibration in modifying shock structure. He studied $O_2$ and $N_2$. A major growth of interest in all of the 'internal' degrees of freedom in molecules emerged rapidly, prompted by the needs of engineers concerned with the design of propulsion systems and re-entry body heat shields for rockets and satellites. Almost concurrently the declassification of work on the containment of plasmas stimulated widespread study of ionizing shock waves and the behaviour of shock waves in plasmas.

Shock waves in polyatomic gases and those in air strong enough to populate internal vibrational states were observed to have a narrow, unresolved shock front followed by a wide, exponentially decaying tail. Of the internal atomic and molecular energy states of gases, only molecular rotation was found to follow the kinetic temperature rapidly; within about four collisions. Vibrational relaxation, dissociation, chemical reactions, and several electronic processes generally required much longer times and larger distances to reach equilibrium. Further developments in the continuum theory of shock structure were thus encouraged, using the models of Landau and Teller based on quantum collision theory and physical chemistry to describe the processes of energy exchange between the internal modes.

This was done for vibrational relaxation by Griffith, Brickl & Blackman (1956), for chemical dissociation by Lighthill (1957, 1960) and for ionization by Petschek & Byron (1957). These models assumed a shock front of negligible thickness to which the Rankine–Hugoniot relations were applied with the specific heat ratio, $\gamma$, calculated using only the fast equilibration of translational and rotational motions. Full equilibrium was reached through a simple exponential relaxation in the case of vibration, and more complex functions for the other processes. One consequence of the success of this type of model was the experimental determination using shock tubes of a large number of rate constants for vibrational relaxation, dissociation, fast chemical reactions and electronic excitation and ionization processes. Theoretical calculations of these rates has continued to prove extremely difficult. Another application has been

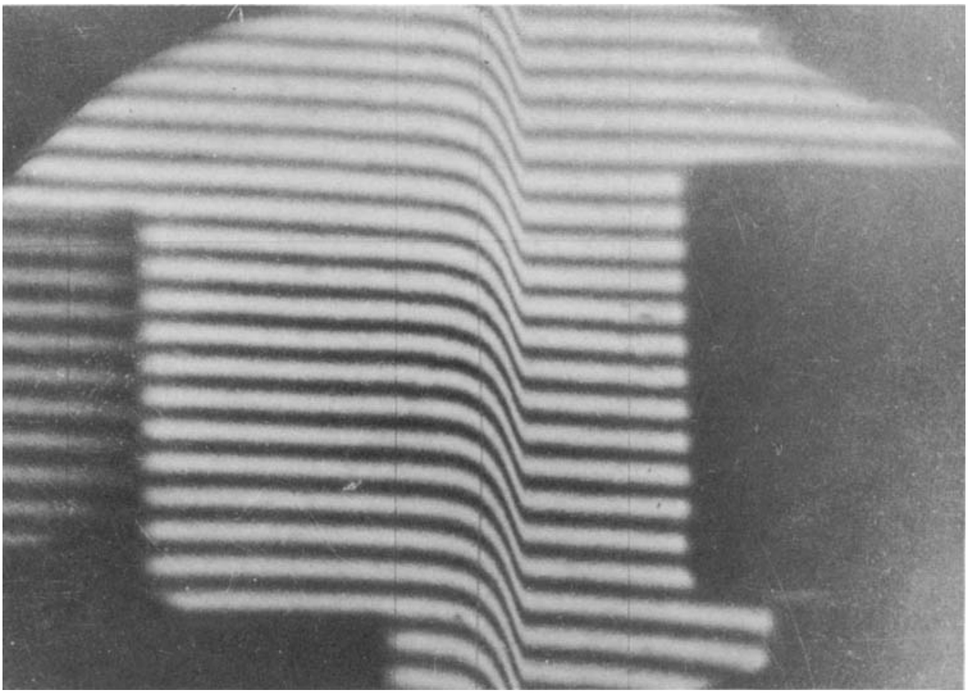

FIGURE 1. Interferogram of a fully dispersed shock wave travelling to the right through $CO_2$ at $M_s = 1{\cdot}04$. Density is proportional to the vertical displacement of the fringes. The two outer hairlines are 1 in apart (Griffith & Kenny 1957).

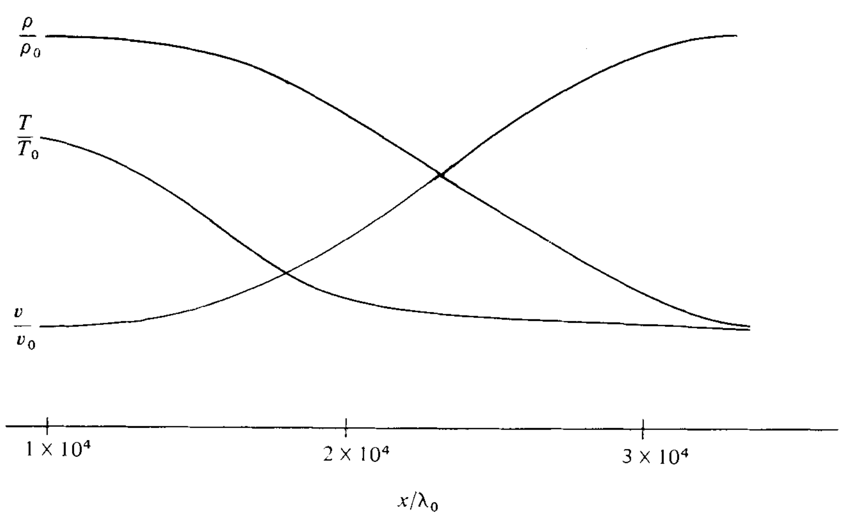


FIGURE 2. Theoretical temperature, velocity and density profiles of a plane shock travelling to the right in a plasma in the presence of a transverse magnetic field. The $X$-dimension is expressed in terms of the upstream mean free path for neutral atoms. The case shown is for magnetic pressure equal to kinetic pressure, low electrical conductivity and a shock density ratio of $1{\cdot}5:1$ (Marshall 1955).

to the understanding of plane gaseous detonation waves, where an ambiguity existed regarding the application of the Chapman–Jouguet condition. Since the speed of sound depends on $\gamma$, the local value varies, depending on whether the fast energy modes or the full equilibrium value is used. This distinction will also prove helpful in resolving a confusing aspect of the theory of Mach reflection discussed later.

The existence of two sound speeds in fluids with lagging internal energy states leads to some interesting physical effects. The first of a number of examples of dispersed shock waves was studied theoretically by Lighthill (1956) and observed experimentally by Griffith & Kenny (1957). For carbon dioxide at room temperature, the slow and fast sound speeds differ by a little over 4%. A disturbance moving with a speed between the two has a profile that is limited by the rate of vibrational excitation and cannot steepen further, as shown in figure 1. An excellent survey of the role of such dispersion mechanisms on the propagation of waves in the atmosphere was made by Johannesen & Hodgson (1979).

Magnetogasdynamic shocks have also been found to be dispersed, in part because the great mass difference between electrons and ions makes energy exchange slow and partly through the addition of a magnetic pressure term. Marshall (1955) found that, for a gas of low electrical conductivity with equal magnetic and kinetic pressure, a shock is spread over $10^4$ collisions as shown in figure 2. In sufficiently rarefied plasmas in the presence of a magnetic field, collisionless shocks have been observed (see DeSilva, Dove & Spalding 1971, for example). Two-body collisions between particles are replaced by longer-range interactions between fields and charged particles; typical dimensions in the compression front being of the order of the ion gyroradius. Such collisionless shocks have been observed in the region where a planetary magnetosphere forms a bow shock in the solar wind.

The collisional structure of shock fronts in simple gases remained an enigma during the long period when these other aspects of complex shocks were being resolved. Liepmann, Narasimha & Chahine (1962) identified the low-pressure side of a shock front up to the point of maximum gradient as the region in which the major theoretical uncertainties existed. A considerable quantity of data on shock front profiles was gradually obtained using improved techniques in measurement and the range of speeds studied was finally extended to $M = 10$ (Alsmeyer 1976). Theoretical work led to a greater appreciation of the importance of the relation between the repulsion–attraction force field assumed for molecules and the temperature dependence of the viscosity coefficient. The proper manner of expanding the velocity distribution function, however, continued to elude theoreticians.

Bird (1970) tried a new approach, a direct simulation of the Boltzmann equation using the Monte Carlo method, with impressive success. Using his results and Alsmeyer's (1976) data, Elliott, Baganoff & McGregor (1977) devised a method for greatly simplifying the computational problem in Grad's thirteen-moment method by some plausible choices for the characteristic temperature and velocity used in the distribution function. Figure 3, from Elliott *et al.*, illustrates the agreement between their computations and Alsmeyer's data. It thus appears that proper theoretical methods for predicting the structure of shock fronts have finally been found. In retrospect the notion of adding successive approximations to the upstream velocity distribution function $f_0$ didn't work well because an unmanageable number of terms would have been needed, as pointed out by Elliott *et al.* Limits to the applicability of

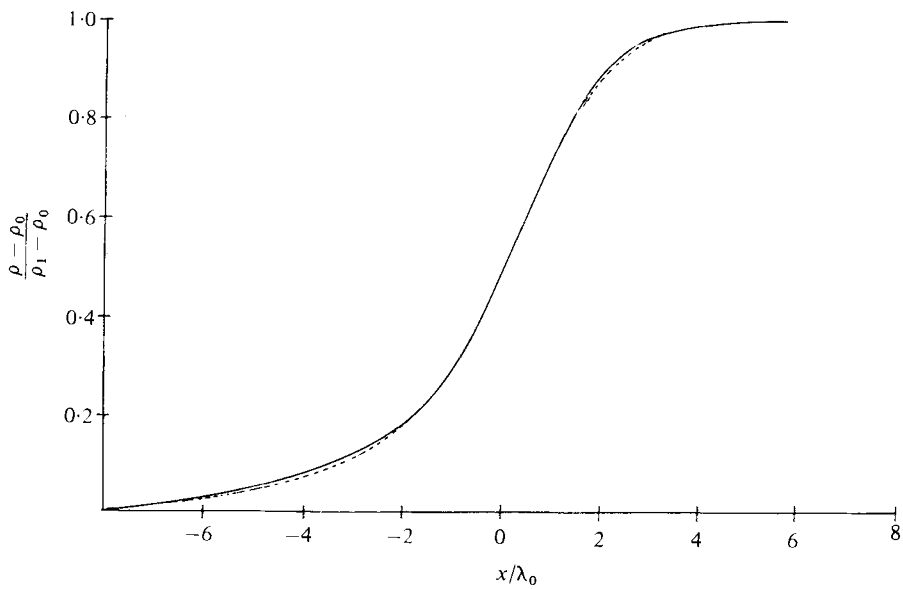


FIGURE 3. Structure of a shock of $M_s = 9{\cdot}0$ in argon. · · · · · ·, Alsmeyer's data; ——, theory of Elliott *et al.* for $\mu \propto T^{0{\cdot}72}$. Theory and experiment show excellent agreement over the central 60 % of density change.

the Navier–Stokes equations to strong shocks found by Liepmann *et al.* result from the same circumstance.

One more facet of shock structure and propagation is of theoretical interest even though absolutely no experimental data are available: relativistic shock waves. These are believed to occur in nova outbursts and other cosmic events and may appear in high-energy collisions of nuclei. Chapline & Weaver (1979) analysed the structure of relativistic shocks in simple gases having values of $\gamma$ between 2 and 10 and, neglecting possible particle production, found no unusual features.

## 3. Regular and Mach reflection

The first recording of shock reflection was made by Mach (1878) using two simultaneous sparks to generate intersecting waves. His experiments were done in the half-plane above a flat surface dusted with lampblack. A trace remained along the path on which shocks intersected, as shown in figure 4. Beyond a certain angle the straight trace split into two curved tracks, indicating that a third shock had formed which connected the initial wavefronts. This interesting but seemingly benign phenomenon has proven to be the focus of one of the most persistent and frustrating pursuits of understanding in fluid mechanics.

The study of Mach reflection, the name eventually given to three-shock reflection, revived during World War II when scientists became concerned with the effects of large air blasts. Depending on angles and distances, a structure might be impulsively loaded by the incident–reflected shock pair or by the single, stronger Mach stem. Field measurements were supplemented by laboratory studies in the shock tube, re-invented to calibrate field pressure gauges. Sketches of typical regular and single

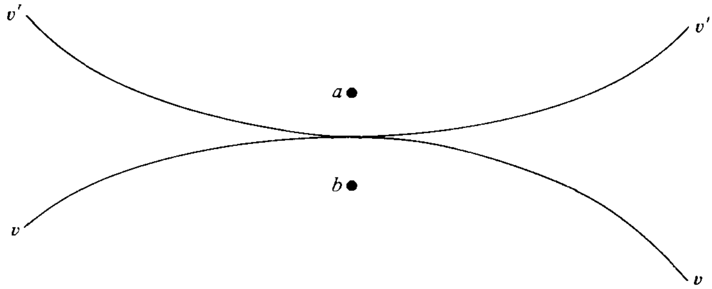


FIGURE 4. Sketch taken from Mach's 1878 drawing of the *Interferenzstreifen vv and v'v'*, showing the *V-förmige Ausbreitung* of the locus of points along which shocks initiated at *a* and *b* intersect. Simultaneous sparks at points *a* and *b* each generated spherical shocks. In travelling over a flat surface dusted with lampblack the shocks left a distinct trace along their points of intersection, starting at the point midway between *a* and *b*, and then advancing along the bisector of *a* and *b*. During this early phase of interaction the spherical shocks reflected off each other as though in regular reflection from an imaginary plane through the bisector. Subsequently a Mach stem formed, connecting corresponding points on curves *vv* and *v'v'*. Since only the locus of points where shocks intersected was recorded, Ernst Mach displayed impressive deductive power in grasping the meaning of traces like this.†

Mach reflection patterns are shown in figure 5, along with the angles defining conditions close to the shock intersection point.

The planarity and extreme thinness of shock waves in air permitted quite accurate measurements of angles to be made from spark shadowgraphs taken with the shock tube. L. Smith (1945) mapped out the regions of regular two-shock and Mach three-shock patterns. He observed that regular reflection occurred for sufficiently low angles of incidence $\alpha$ (or high wedge angles $\theta_w$) at all shock strengths. He also determined the boundary between regular and Mach patterns by extrapolating data on the relative height of the Mach stem, as measured by the angle $\chi$, to zero.

A theoretical analysis by von Neumann (1943; see von Neumann 1963) was made using the following assumptions:

(1) The fluid is an inviscid perfect gas with constant $\gamma$.

(2) Shocks are discontinuities with finite curvature.

(3) Close to the shock intersection, two-dimensional steady-flow theory is applicable.

(4) The net flow deflection in regular reflection is zero; $\delta_1 + \delta_2 = 0$.

(5) For three-shock reflection, downstream pressures and flow angles behind the incident–reflected shock pair and the Mach stem are equal. This implies the existence of a slipstream which separates flows of differing speeds and entropies.

The two-shock theory yielded two solutions for the reflected wave and the three-shock theory gave multiple solutions for sufficiently strong incident shocks. Comparison with experiment confirmed Nature's choice of the weaker of the two possible reflected shocks in regular reflection, so von Neumann used the weaker shock argument in selecting the physically correct solution from the multiple values yielded by three-shock theory.

Since even these solutions gave regions in $\gamma, \alpha, M_0$ space where both two- and three-shock patterns might occur, criteria for the transition were needed. One likely choice for the limit to regular reflection was the extreme condition on flow deflection by the reflected shock where the weak and strong shock solutions converge. In steady

† I thank Professor G. T. Reynolds and Dr Peter Cziffia of Princeton for providing a copy of Mach's original paper.

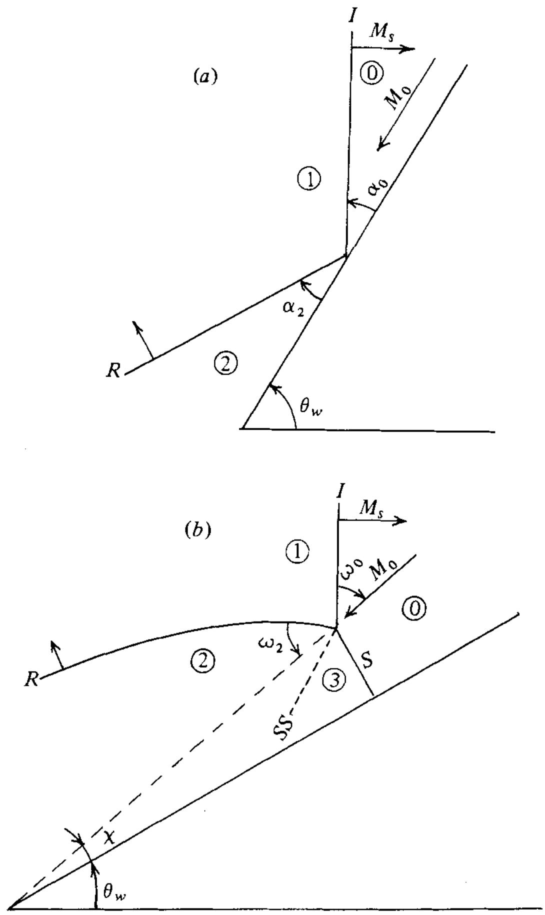


FIGURE 5. Typical regular ($a$) and single Mach reflection ($b$) patterns in pseudo-stationary flow. The incident shock $I$ of speed $M_s$ reflects from a wedge of inclination $\theta_w$, producing a reflected shock $R$. For sufficiently high angles of incidence $\alpha_0$, a Mach stem $S$ and slipstream $SS$ appear. In a frame of reference moving with the shock intersection point, $M_0 = M_s/\sin(\theta_w+\chi)$, and $\omega_0 = \alpha_0 - \chi$. The latter variables describe the comparable steady-state reflection observed in a wind tunnel.

aerodynamics this is the half-wedge angle at which a bow shock in supersonic flow becomes detached with maximum flow deflection being $\delta_D$ and the corresponding shock angle designated as $\alpha_D$. Von Neumann also considered another criterion that he called direct pressure compatibility. With this concept, transition would occur when a vanishingly small Mach stem normal to the incoming flow could link the intersection of incident and reflected shocks to the wall with equal pressures behind. Such patterns can exist only for incident shock strengths above a limiting value dependent on $\gamma$. For $\gamma = 1{\cdot}4$ he found this shock pressure ratio to be 2·3086, corresponding to $M_0 = 2{\cdot}2$. The resulting condition for transition gives a value for incidence angle $\alpha_N$ that is less than $\alpha_D$. Von Neumann concluded that $\alpha_D$ was the appropriate transition criterion for weak shocks ($M_0 < 2{\cdot}2$), and $\alpha_N$ for strong shocks ($M_0 > 2{\cdot}2$).

Shock-polar diagrams for two- and three-shock processes were used by Kawamura & Saito (1956) to study the flow conditions for which von Neumann's transition

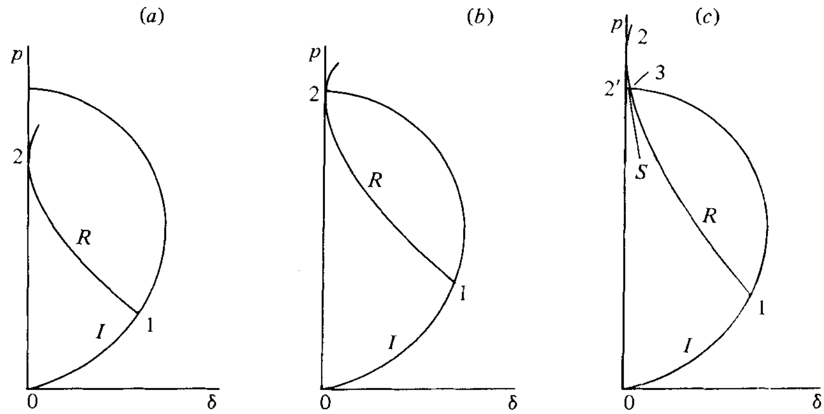


FIGURE 6. Criteria for transition from regular to Mach reflection in shock-polar plots. $I$ is the incident shock, $R$ the reflected. (a) The case where the reflected polar becomes tangent to the pressure axis below the $I$-polar intercept. (b) Von Neumann's condition where $\alpha_0 = \alpha_N$ and transition occurs with an infinitesimal Mach stem. (c) At still higher values of $\alpha$ two shock patterns are possible, 0–1–2 and 0–(1, 2′)–3.

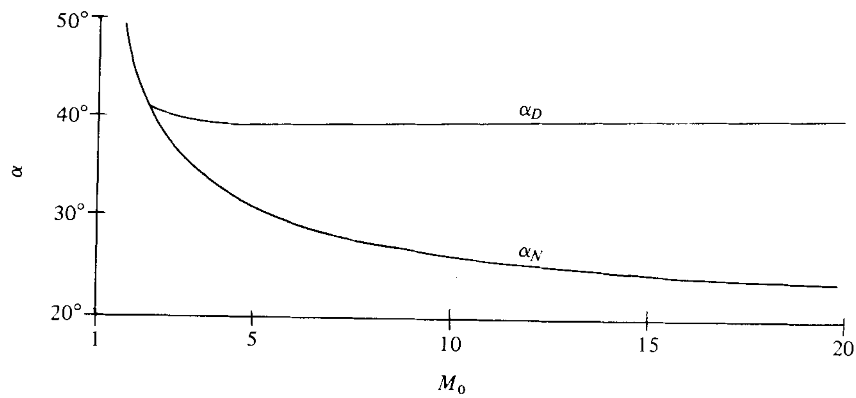


FIGURE 7. Theoretical curves for gases with $\gamma = \frac{7}{5}$ showing the rapid divergence of the two criteria for transition, $\alpha_D$ and $\alpha_N$, at shock Mach numbers above $M_0 = 2{\cdot}2$. For gases with $\gamma = \frac{5}{3}$, the separation is slightly less. Shock-tunnel data show clearly that $\alpha_N$ is the proper criterion in steady flows, while transition occurs somewhere between $\alpha_N$ and $\alpha_D$ for $M_0 > 2{\cdot}2$ in pseudo-stationary flows.

criterion change from $\alpha_D$ to $\alpha_N$. The three polar plots in figure 6 illustrate possible limits to two-shock reflection for $M_0 \gtreqless 2{\cdot}2$ for gas with $\gamma = 1{\cdot}4$. For weak incident shocks, $I$ compresses and deflects the flow from state 0 to state 1. The reflected shock polar $R$ intersects the pressure axis below the $I$-polar intercept. The solution for maximum deflection by the reflected shock arises when the reflected shock polar is just tangent to the $p$ axis as shown in figure 6 (a), taking the gas from state 1 to state 2. Figure 6 (b) shows the situation when $M_0 = 2{\cdot}2$, the lowest speed at which von Neumann's condition of direct pressure compatibility can be satisfied. For $M_0 > 2{\cdot}2$, the $I$ and $R$ polars are shown in figure 6 (c), again for the solution in which the $R$-polar is tangent to the $p$ axis. Point 2 still corresponds to the condition of maximum deflection, $\delta_D$, by $R$, but a three-shock solution at point 3 is also possible. The Mach stem $S$

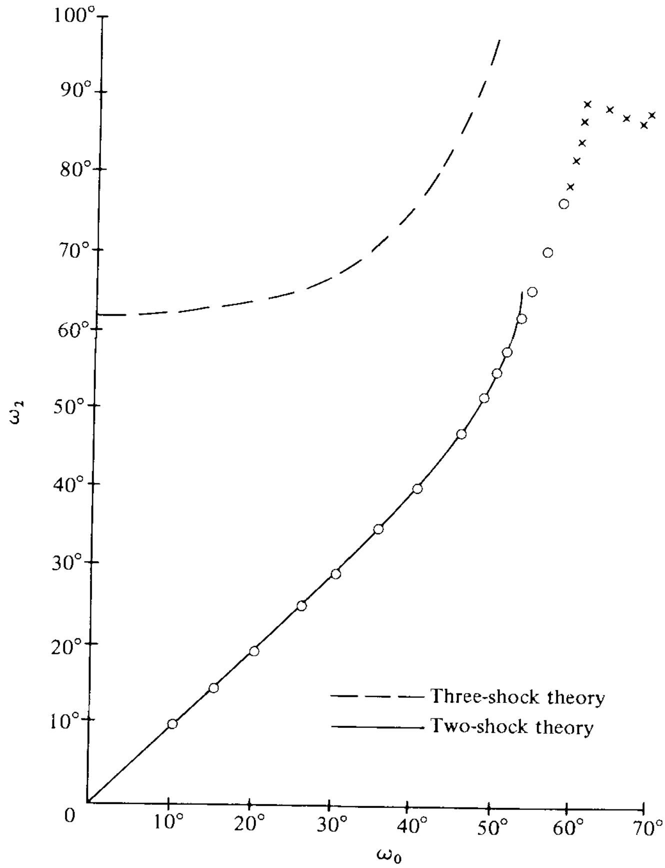


FIGURE 8. Experimental data of L. Smith and theory for two- and three-shock patterns for an incident shock of inverse pressure ratio 0·8, $M_s = 1{\cdot}1$. Both the slight departure from two-shock theory and the persistence of regular reflection beyond the theoretical limit have been re-affirmed from recent experiments by Henderson. ○, Regular reflection; ×, Mach reflection. (Bleakney & Taub 1949.)

provides equal pressures and flow deflections behind the triple point where $I$, $R$ and $S$ meet. According to von Neumann, the three-shock pattern should be observed under these conditions rather than the two-shock solution. With increasing incident shock speed $\alpha_N$ and $\alpha_D$ diverge appreciably as shown in figure 7.

Smith's data showed agreement between theory and experiment for both two- and three-shock patterns at the higher incident shock strengths studied. For incident shocks of pressure ratio 1·25, however, the deviations from both two- and three-shock theories reproduced in figure 8 appeared. In view of the agreement noted above, the clear departure from theory in regular reflection, the apparent persistence of regular reflection beyond $\alpha_D$ and the unconvincing gap in three-shock reflected angles was particularly puzzling. Other theoretical approaches were tried and different transition criteria suggested, mostly to little avail. We shall see that von Neumann's criteria are most likely the correct (or nearly correct) ones although it has taken thirty-five years to find this out. Meanwhile the disparity between theory and experiment became known as von Neumann's paradox.

Additional data on the transition from two-shock to three-shock configurations seemed only to deepen the paradox. Besides single Mach reflection two additional patterns were found at high incident shock strengths. Now commonly referred to as complex Mach reflection, first observed by L. Smith (1945), and double Mach reflection, first observed by White (1952), both are associated with an outflow behind the reflected shock that is supersonic relative to the triple point. These patterns are peculiar to time-dependent flows in a shock tube; steady wind-tunnel data shows only regular or single Mach reflection patterns. The cause of the additional patterns in shock tube experiments has been traced by Gvozdeva *et al.* (1969) to an interaction between the main flow over the wedge and vibrationally excited gases coming from the reflected shock and Mach stem with a correspondingly increased specific heat ratio. Ben-Dor & Glass (1979) have mapped out the regions in $M_s, \theta_w$ space for single, double and complex Mach reflection.

The equivalence between steady shock patterns seen in a wind tunnel and an observable region around the point of shock intersection in pseudo-stationary shock tube flow has been carefully considered. Bleakney found no discernible departure from pseudo-stationarity over a time factor of about 50: 1 for single Mach reflection. On the other hand parts of the flow fields in complex and double Mach reflection are neither steady nor pseudo-stationary and, as mentioned before, have no counterpart in steady wind-tunnel flows. Sternberg (1959), Zaslavskii & Safarov (1973), Henderson & Lozzi (1975), Hornung, Oertel & Sandeman (1979) and others have searched for 'hidden' departures from the locally steady patterns actually observable, with ambiguous results at best.

Recent measurements made with careful attention to obtaining good optical resolution of nearly two-dimensional flows in wind tunnels and shock tunnels have cleared up at least a part of the long-standing confusion over the correct transition criteria for strong incident shocks. For steady flows at $M_0 = 3$ and 4 the data of Henderson & Lozzi (1975) and that of Hornung *et al.* (1979) up to $M_0 = 15$ convincingly support $\alpha_N$ as the correct criterion. As indicated by figure 7, $\alpha_D$ and $\alpha_N$ are well separated at these shock speeds. No such clear-cut statement can be made for the pseudo-stationary case. The data of different investigators show regular reflection to occur in the zone between $\alpha_N$ and $\alpha_D$, but do not firmly establish where it ends.

One aspect of Mach reflection that has given experimentalists particular trouble is determination of the angle $\chi$, giving the locus of the triple point in pseudo-stationary flows. This angle is not predicted by the local theory. The Mach stem and its most readily observable indicator, the slipstream, become lost to view close to the inclined wedge for Mach reflection near transition. W. R. Smith (1959) eliminated the optical interference of the inclined wedge by observing the interaction in free space between shocks reflected from a pair of tilted plates. His data for weak shocks, $M_0 < 2{\cdot}2$, is probably the best available and supports von Neumann's conjecture that Mach reflection occurs for $\alpha > \alpha_D$. Ben-Dor & Glass (1980) and Henderson (1980) compare approximate theories for $\chi = \chi(\theta_w, M_0)$ with their experimental data and do not find completely satisfactory results (figure 9). The theoretical prediction of $\chi$ or Mach stem length evidently awaits a full-field solution for pseudo-stationary flow.

Because no adequate solution for the entire transient flow field has been devised, experimentalists are constrained to compare their observations with a local, steady-state theory. This presents a quandary; each data point represents the flow pattern at an instant of time and for predetermined initial conditions. There is no physical

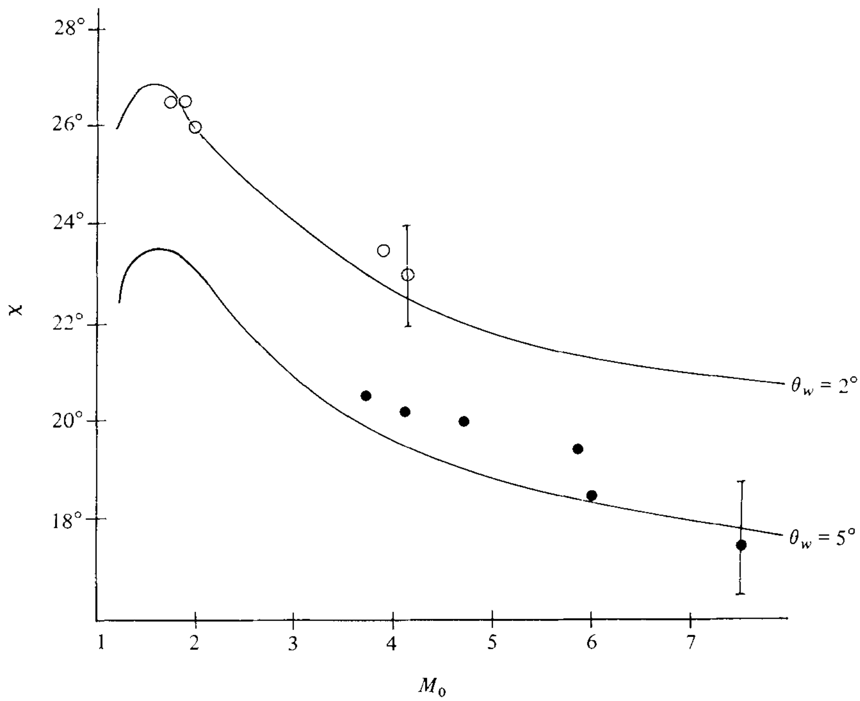


FIGURE 9. Ben-Dor's (1978) approximate theory and data for the angle $\chi$ (see figure 5) for small wedge angles. As $M_s \to 1$, $\chi \to 0$.

connection between the flow pattern in one experiment and another having slightly altered initial or boundary conditions. Any differences observed cannot reasonably be taken to imply a continuous change between the two. In some very interesting work by Hornung *et al.* (1979) a length-scale effect is suggested to explain conflicting data in the range between $\alpha_N$ and $\alpha_D$. They argue that, since the generation of a Mach stem introduces a meaningful length scale into an otherwise dimensionless regime, some signal must be able to reach the point of shock intersection to maintain Mach reflection. This strikes me as spurious reasoning for pseudo-stationary flows because an observed pattern showing a small Mach stem is a completely independent event from one showing regular reflection at a slightly smaller $\alpha$. Each pattern has presumably grown from the time the incident shock first reached the inclined wedge, at point 0 in figure 5, so no change in pattern need occur. If concave or convex wedges are used so that $\alpha$ varies between $\alpha_N$ and $\alpha_D$, then the entire process is transient and a hysteresis effect may well arise. Ben-Dor, Takayama & Kawauchi (1980) and Itoh, Okazaki & Itaya (1981) indeed report such a transient effect.

Thus far the discussion has focused on the criteria for transition between two- and three-shock reflection patterns and the presumption that observers have been able to resolve flow patterns near enough to shock intersections for angle measurements to represent those of a locally steady region. We see that von Neumann's first assumption, of an inviscid perfect gas with constant $\gamma$, has scarcely been examined. He recognized the dependence of wave angle on $\gamma$ and tabulated critical conditions for a number of values of $\gamma$. Extensive computations for reflected shock angles were made only for $\gamma = 1{\cdot}400$, however, and for years experiments in air were compared with these values.

Only recently have some of the most important departures from such idealized gas behaviour received quantitative treatment.

Almost all real-gas effects act to increase the true values of $\alpha_N$ and $\alpha_D$ or otherwise deceive observers into thinking their data shows regular reflection persisting into a range where Mach reflection should theoretically occur [based on $\gamma = 1{\cdot}400$]. These include vibrational contributions to the specific heat and time-dependent dissociation of oxygen and nitrogen. Also, the displacement thickness of the boundary layer on the wedge behind an incident shock is negative for a frame moving with the shock confluence point. Ben-Dor & Glass (1979) and Hornung *et al.* (1979) present real-gas computations and the latter estimate displacement effects. Many of the earlier discrepancies and inconsistencies seem explainable with these factors, but no one has been able to explain why the idea of a locally steady flow model is not applicable for weak incident shocks.

That the clarity and elegance of von Neumann's early theoretical ideas on shock reflection still pervade thinking about the problem is evident from the preceding discussion. Also apparent are the observations that he was correct in identifying the physically observable branch of multivalued solutions and the criteria for appearance of two- and three-shock patterns. Two significant theoretical problems remain as residuals of the 'von Neumann paradox': the possibility that two stable configurations exist between $\alpha_N$ and $\alpha_D$ for strong shocks in pseudo-stationary flows and a model for the Mach-like shock patterns observed by White (1952) for $M_0 < 1{\cdot}25$. Henderson (1964) showed that no solutions of von Neumann's equations exist for $M_0$ in this range when the flow deflections by incident and reflected shocks have opposite signs. Ben-Dor (1978) found solutions for $1 < M_0 < 1{\cdot}25$ when $I$ and $R$ both deflect the flow towards the wall but experiments don't indicate the existence of the necessary forward-facing inclination of the reflected shock $R$.

## 4. Refraction

The discussion in this section will address the interaction of a plane shock incident at an angle $\alpha$ upon a discontinuity in some thermodynamic property that changes the acoustic impedance $\rho a$, where $\rho$ is density and $a$ is the local sound speed. Even for this simple, restricted case the great variety of refraction patterns found is indicative of the complexities of both civil and military problems. Civil applications include geophysical explorations using underground explosions and the sonic bang from supersonic aircraft. Military applications include refraction of shocks from air and underwater bursts.

Volume 1 of this Journal contains the first comprehensive experimental study of shock refraction, by Jahn (1956). He worked with pairs of gases having the same pressure and temperature, separated by the thinnest membrane he could fabricate. A number of steady, pseudo-stationary, and irregular transient patterns of interaction were found. A theory for the locally steady flows had been worked out by Taub (1951) and Polachek & Seeger (1951) using a model similar to that employed by von Neumann in analysing regular and Mach reflection. A substantial extension to identifying possible irregular patterns and the conditions for which they might occur was made by Henderson (1966) and a general definition of shock impedance developed by the same author (Henderson 1970). Abd-el-Fattah, Henderson & Lozzi (1976) and Abd-el-

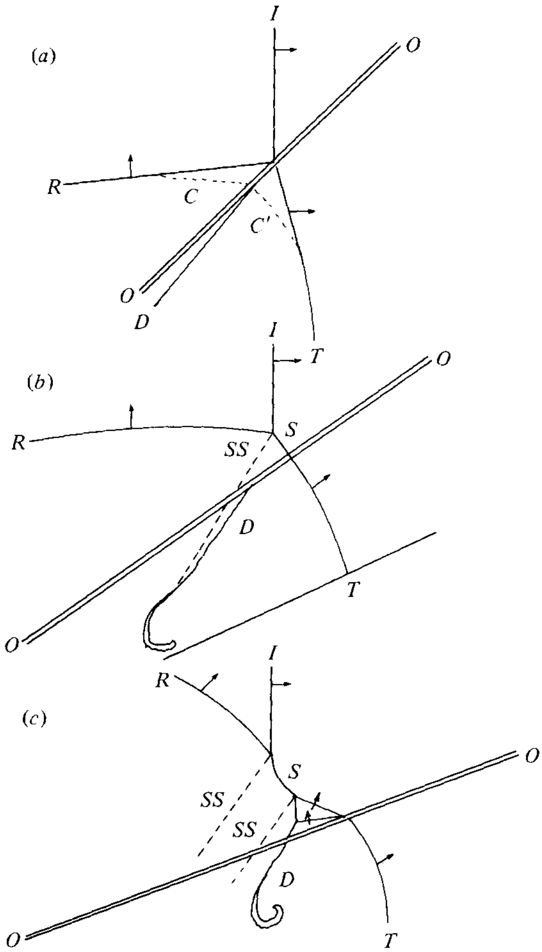


FIGURE 10. Some patterns of shock refraction. (*a*) Regular refraction at an air–methane interface, line $O$–$O$. The incident, transmitted and reflected shocks $I$, $T$ and $R$ are straight until caught by the respective sonic corner signals $C$ and $C'$. The interface is deflected to $D$ (Jahn 1956). (*b*) Irregular refraction at an air–methane interface showing a Mach stem $S$, slipstream $SS$, as well as the features identified in (*a*) (Jahn 1956). (*c*) Irregular refraction at a carbon dioxide–methane interface showing a precursor in the methane. From Abd-el-Fattah & Henderson (1978*b*), who mapped out the boundaries between numerous types of irregular patterns.

Fattah & Henderson (1978*a*, *b*) experimented with more disparate gases and produced extensive data for several regimes.

Three representative patterns are shown in figure 10, showing regular refraction with straight shocks near the interaction point, a pseudo-stationary interaction with a scale length introduced by a Mach stem, and an irregular transient flow where the transmitted shock races ahead of the incident shock to generate a precursor. We note that the approach to ideal conditions is inherently less complete for refraction experiments than for the reflection studies discussed previously. The inertia and stiffness of the membrane and edge effects introduce complexities that are difficult to quantify. Nevertheless some general conclusions may be drawn. Where theory gives multiple values for the strength of the transmitted shock, the weakest one is always observed

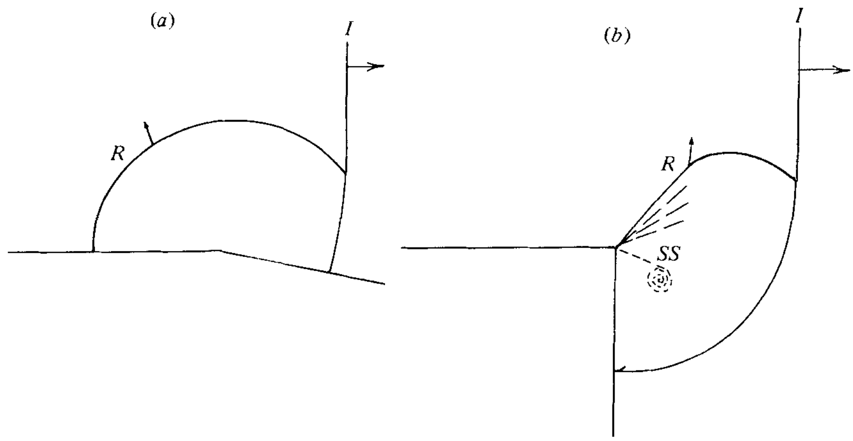


FIGURE 11. Diffraction at a corner: (*a*) weak shock and small corner deflection, (*b*) strong shock and 90° corner. No satisfactory theory exists for predicting where the Prandtl–Meyer expansion ends or for the strength of the diffracting shock (Griffith & Brickl 1953).

(Henderson & Macpherson 1968). In the region of regular refraction acceptable agreement exists between the locally steady theory and experiment, and transition to a more complex pattern occurs when either the deflection condition $\alpha_D$ is reached or some wave becomes sonic.

From the viewpoint of current practical interest most shock refraction problems involve extended regions with finite gradients rather than discontinuities in some fluid property. Examples of the latter may however be mentioned. One is the role of precursors in altering strong air shocks moving over water. Another is the propagation of finite-amplitude acoustic waves across the annular regions of differing Mach number behind a three-spool jet engine. Henderson & Macpherson (1968) have shown that refraction at a flow Mach number interface and a gaseous interface are qualitatively identical. If the physical patterns also retain their underlying identities then some of the insights gained may apply to refraction at diffuse interfaces as well.

## 5. Diffraction

The complement of reflection at a concave corner, diffraction of a plane wave over a convex corner, was originally thought to be an especially tidy problem for analysis because the region of disturbed flow is contained by the solid wall, the incident shock and a Mach circle (with the addition of a Prandtl–Meyer fan from the corner for strong incident shocks). The original questions of interest were whether solutions would shed any light on the von Neumann paradox or not and the extent of weakening of the incident shock as a function of corner angle.

Figure 11 illustrates two cases: a relatively weak shock diffracting at a small corner angle and a strong shock diffracting around a 90° corner. A linearized theory for small corner angles and weak shocks was developed by Lighthill (1949). This gave excellent agreement with observation but for small concave corners failed to clarify the Mach reflection problem because the method assumed vanishing strength for the reflected wave near the triple point.

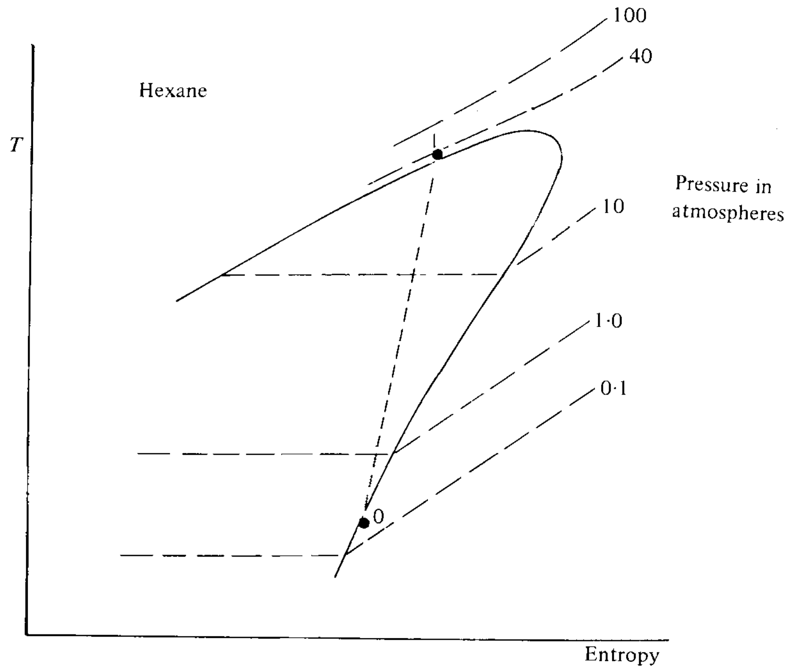


FIGURE 12. Temperature–entropy diagram of the saturation region for hexane showing how a shock wave converts pure vapour to pure liquid.

For large corner angles at all incident shock strengths a slipstream grows from the corner at an angle corresponding approximately to the initial pre-shock pressure. This slipstream wraps up into a structured vortex that may be accompanied by an embedded shock for the faster flows. The incident shock grows in a pseudo-stationary manner. It is observed to have appreciable strength throughout its length. Even a 180° turn fails to provide immunity from blast effects. Skews (1967) and Bazhenova *et al.* (1971) made numerous measurements with incident shock Mach numbers up to 4·5 and 10, respectively, and observed a wide array of effects for which no theory has been proposed.

One particularly intriguing aspect of shock diffraction is the early appearance of a slipstream and vortex. Clearly originating through the action of viscosity, the effect has been detected by Emrich & Reichenbach (1969) only 2 $\mu$s after passage of an incident shock over a corner. The diffraction of a shock over an airfoil at angle of attack produces both transient local forces and a sudden net change in lift. The corresponding change in circulation about the airfoil is balanced by that around a vortex which is shed into the wake as the trailing edge slipstream rolls up. The similarity of this problem to gust loading of airfoils in flight has been recognized but not pursued quantitatively.

## 6. Shock waves in liquid helium I and II

At temperatures below 2·17 K liquid helium forms a superfluid component, LHeII, that can move without interaction through normal LHeI. Two independent waves have been observed in LHeII associated with the transfer of momentum (pressure) and energy (temperature), called first and second sound. Their properties have been studied experimentally and theoretically in the linearized approximation but only recently has attention focused on nonlinear effects.

Using a vertical cryogenic shock tube developed by Liepmann, Cummings (1976) investigated the propagation of strong shocks in LHeI and LHeII at 2·62, 2·02, 1·91, 1·71 and 1·46 K. The incident shock passed vertically downward through helium vapour to strike a vapour–liquid interface. Since the impedance match between these two phases is quite good in the case of helium, adequate signal strength and time resolution were obtainable to follow both transmitted and reflected waves.

Below the $\lambda$-transition temperature of 2·17 K the single shock front observed in LHeI was replaced by two shocks transmitted through the liquid, corresponding to first and second sound pulses. Their speeds differed by an order of magnitude. In general the quantitative values obtained for first-sound shock waves were within a few per cent and second-sound shocks about 25% faster than predicted by available theory. Further experiments may elucidate first–second sound shock interactions and provide useful guidance in the theoretical study of nonlinear effects in superfluidity. This set of phenomena is the only known instance where elementary quantum processes lend themselves to direct observation by conventional shock-wave techniques.

## 7. Condensation and liquefaction shocks

The appearance of a mixed vapour–liquid phase or even a pure liquid behind a shock front may result from specially contrived circumstances. Although the theory of shock formation precludes rarefaction shocks in general, compression shocks in a vapour that has been supercooled by rapid expansion yield a partial condensation. This phenomenon has been known for a long time in supersonic wind-tunnel work, where water vapour condensation places a limit on tunnel operations. A similar effect in hypersonic tunnels necessitates heating of the reservoir supply to stay above the saturation region of the working fluid. Without heating, expansion nozzles have been used to study nucleation processes; see, for example, Wegener & Wu (1976) and Glass, Kalea & Sislian (1977).

Plane shocks with a pure liquid phase on the downstream side have recently been observed by Dettleff *et al.* (1979). With both upstream and downstream states in thermodynamic equilibrium, this phenomenon depends on finding a material with very high specific heat and a saturation region having a positive slope on the vapour side in the temperature–entropy plane. Figure 12 shows hexane $C_6H_{14}$ to have the latter property. Its specific heat is $\sim 24R$, just sufficient to meet the limitation on shock temperature rise prescribed by theory for a liquefaction shock. It may be seen from the figure how a shock produces a pure liquid in principle.

In practice Dettleff and his co-workers used commercial fluorocarbons and, because the liquid densities are so large compared to vapour densities, arranged their experiment so that the expected liquid phase would be formed behind a shock reflected from the end of their shock tube. Their data generally support the predictions of classical shock-wave theory for the final state. Photographs show small spots throughout the liquid which the authors suggest are vortices formed by the differential motion of droplets formed during the early stages of condensation.

## 8. Numerical methods and other approximations

The reader may find the heading of this section incongruous after its predecessors which deal in turn with various elementary shock processes. For explanation consider two important practical problems: raindrops impinging on the bow shock of a vehicle in supersonic flight, and shock propagation through a channel of varying area. These are instances of external and internal flows, respectively, that involve reflection, refraction and diffraction. In principle they could be solved using information available from the preceding sections. In practice we are sometimes concerned not with the detailed interactions, but just with the overall effects that result.

Several conceptual approaches have been proposed for reducing the inherent complexity of analysing flows involving multiple shock-wave interactions. The objectives have been either to improve intuitive knowledge about processes or to obtain solutions of useful accuracy with acceptable costs in time and effort. A comparison of the results of such methods with complete theory, if available, and with precise experimental results for some simple configurations tests the range of useful application and accuracy. In this section such methods are reviewed for their utility in treating the basic shock interactions discussed earlier.

The acoustic approximation is historically the oldest and still the most widely used method for getting around the nonlinear terms in the flow equations. With this approximation, regular reflection is described as a plane wave with a small pressure rise $\delta p$ striking a surface at angle of incidence $\alpha$, followed by a reflected wave also having a pressure rise of $\delta p$ and with the angle of reflection $\alpha' = \alpha$. The resulting total pressure rise at the surface is accordingly $2\delta p$. A paradox immediately arises: for all angles of incidence $\alpha \neq 90°$, the pressure rise is $2\delta p$, while, for $\alpha = 90°$ exactly, the pressure rise must be just $\delta p$. The paradox is removed when second- and third-order terms are considered, for then the reflected wave pressure and angle both change for any finite $\delta p$, while for a finite departure of $\alpha$ from $90°$ a Mach-like reflection occurs. Thus regular reflection can occur at all angles of incidence but its realm approaches the $M = 1$, $\alpha = 90°$ corner along the $M = 1$ $(\delta p = 0)$ axis.

G. A. Bird has explored the utility of modelling flows by simulating the motions of thousands of molecules with a computer. Auld & Bird (1977) applied this Monte Carlo method to two- and three-shock reflections, but could not obtain adequate resolution of the flow features to state where the transition to Mach reflection occurs.

Full-field computer codes have also been applied to the problem of shock reflection. Schneyer (1975) used a two-dimensional Eulerian code to simulate the time history of an $M_s = 2$ shock striking a wedge of slope 2:1 (regular reflection) and 1:2 (single Mach reflection). He also worked the second problem, using a Lagrangian code for comparison of accuracy and cost. Both pressure and density field were plotted.

The shock fronts were necessarily smeared out over a few cells by the artificial viscosity terms employed in the schemes, but the positions and strengths of the waves agreed well with prior data. Only the Lagrangian method showed a slipstream emanating from the triple point. Schneyer observed that the Eulerian code was easier to run and cheaper, for the Lagrangian code required rezoning of the grid in regions of high shear.

Kutler & Shankar (1977) reviewed all the available computational methods and reworked the regular reflection problem at $M_s = 2$ and also at $M_s = 4{\cdot}71$. Ben-Dor & Glass (1978) then carried out experiments at the same conditions to provide accurate wave positions and density fields for comparison with both Schneyer's and Kutler & Shankar's computational results. They concluded that the latter results were superior but pointed out that all of the numerical methods gave poor predictions for the detailed density profile along the wall and the region inside the reflected shock.

Book *et al.* (1980) employed a method for minimizing the effects of numerical diffusion within computational cells called Flux-Corrected Transport. When applied to shock reflection from a wedge under conditions where double Mach reflection would be expected, striking agreement with the observed pattern of shocks resulted and portions of the computed density field also showed good quantitative agreement with data from interferograms. It appears that computational methods are becoming sufficiently refined to resolve some of the inner details of fundamental concern in basic shock interactions. At all events these methods serve well for handling the overall effects in complex, practical situations of the sort mentioned at the beginning of this section. Computational Fluid Dynamics, the name currently used to identify the family of computer-based methods, is developing so rapidly that some pleasant surprises are certainly in store.

One further method for analysing shock interactions has the beauty of simplicity and an apparent range of uses not wholly explored yet. Often referred to in the literature as the CCW methods, it is based on a series of papers by Chester, Chisnell and Whitham published in the 1950s. In his book, Whitham (1974) shows how the 'rule' is developed and illustrates its remarkable accuracy when applied to several problems that would otherwise be extremely hard to handle. He comments that, although the rule works well in many cases, 'no really satisfactory explanation of this was found'.

Use of the rule to investigate laser-generated shock waves was first made by Jumper (1978). The existing analytical solutions were applicable only to constant power input or to the case of instantaneous energy release at $t = 0$, while numerical computations for a transient power input proved complicated and costly. Jumper found a fairly simple, direct relation between the rate of net energy input and the acceleration of the shock wave. The results agree very well with experiment and are useful in identifying where the steady-state solutions are valid.

In perspective it appears that many formidable problems can be handled with one or another of the methods developed for analysing interacting shock waves. The discrepancies in detail between the results of careful observation and theory are disturbing where they arise, for we are still ignorant of *why* a particular model isn't correct when it should be, and vice versa. As a precaution against being misled in the future when more powerful and complex numerical methods are developed for the practical solution of very complex flow problems, investigators will be well advised to test their methods on some of the simple, basic shock processes for which accurate measurements are available.

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